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CCF

Reg. No. : ....

Name : ....

Combined First and Second Semester B.Tech. Degree Examination, May 2015
(2013 Scheme)
13.101 : ENGINEERING MATHEMATICS – I
(ABCEFHMNPRSTU)

Time : 3 Hours Max. Marks : 100

PART – A

Answer **all** questions. **Each** question carries **4** marks.

1. Evaluate $\lim_{x\to 0}\left[\frac{x-\tan x}{x^3}\right]$.

2. Change the order of integration in the following integral and evaluate $\int_0^{4a}\int_{x^2/4a}^{2\sqrt{ax}} dxdy$.

3. Find the inverse Laplace transform of $\frac{s}{s^4+4a^4}$.

4. Determine whether the vectors (1, 3, 2), (5, – 2, 1) and (– 7, 13, 4) are linearly independent.

5. Using Cayley Hamilton theorem, find A^{-1}, if

$$A=\begin{bmatrix}1 & 2 & -2\\ 2 & 5 & -4\\ 3 & 7 & -5\end{bmatrix}$$

P.T.O.

PART – B

Answer **one full** question from **each** Module. **Each** question carries **20** marks.

Module – I

6. a) If $y = \left(x + \sqrt{x^2+1}\right)^m$, show that

$$(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0.$$

b) If $u = \frac{xy}{z}, v = \frac{yz}{x}, w = \frac{zx}{y}$, find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.

c) If $u = \sin^{-1}\left[\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}\right]$, show that $\frac{\partial u}{\partial x} = \frac{-y}{x}\frac{\partial u}{\partial y}$.

7. a) Find the evolute of the hyperbola $x = ct$, $y = \frac{c}{t}$.

b) A rectangular box open at the top is to have a volume 32 cubic feet. Find the dimensions of the box requiring least material for its construction, using Lagrange's multiplier method.

Module – II

8. a) Evaluate $\iint_A x^2\,dxdy$, where A is the region in the first quadrant bounded by the hyperbola $xy = 16$ and the lines $y = x$, $y = 0$ and $x = 8$.

b) Calculate the volume of the solid bounded by the surfaces, $z = 0$; $x^2 + y^2 = 1$; $x + y + z = 3$.

9. a) Change into polar co-ordinates and evaluate $\iint \sqrt{a^2 - x^2 - y^2}\,dxdy$ over the semicircle $x^2 + y^2 = ax$ in the positive quadrant.

b) Find the area lying between the parabola $y = 4x - x^2$ and the line $y = x$.

Module – III

10. a) Find the Laplace transform of the rectified semi wave function defined by

$$f(t) = \sin wt,\ 0 < t < \frac{\pi}{w},$$

$$= 0,\ \frac{\pi}{w} < t < \frac{2\pi}{w};\ \text{where } f\left(t + \frac{2\pi}{w}\right) = f(t).$$

b) Solve the following differential equation by Laplace transform;

$(D^2 - 1)\,x = a \cosh t,\ x(0) = x'(0) = 0.$

11. a) By using method of variation of parameters, solve $(D^2 + 4)\,y = 4\sec^2 2x$.

b) Using convolution theorem, find the Laplace inverse transform of $\frac{1}{s^2(s+1)^2}$.

Module – IV

12. a) Find the rank of the matrix $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$.

b) Determine K so that the equations $kx + y + z = 1;\ x + ky + z = 1;\ x + y + kz = 1$ may have

i) a unique solution

ii) more than one solution

iii) no solution.

13. a) Reduce the quadratic form :

$10x^2 + 2y^2 + 5z^2 + 6yz - 10xy - 4xy$ to a canonical form by orthogonal reduction and examine for definiteness.

b) Diagonalise the matrix $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ and hence find A^4.
